Statistics Assignment 2 - John Sinclair - 16325734

Q1.

1. 6 x 6 x 6 = 216

A single dice being rolled three times is the same as rolling three dice at once, so with each dice there are 6 permutations, so with two dice the total number of elements in the sample space is 6 x 6 = 36, roll the third and we multiply by 6 again = 216 elements in the sample space.

1. (3C1 \* 5 \* 5) + (3C2 \* 5) + 3C3 = 91 => 91 / 216 = 0.421

To get the number of permutations we must add the probability of getting one 2 plus two 2’s plus three 2’s.

To get the probability of getting one 2: 3C1 \* 5 \* 5 = 75 -this is because choose gives us the number of ways the 2 can be positioned in the sequence, and the other two dice can be any number excluding 2 (5 options each)

Using the same logic we get:

The probability of getting two 2s: 3C2 \* 5 = 15

The probability of getting three 2s: 3C3 = 1

Then divide the number of permutations by the total number of elements in the sample space, hence 91 / 216.

1. Code

rolls = 10000000;

count = 0;

for i = 1:rolls

first = randi(6);

second = randi(6);

third = randi(6);

if (first == 2 || second == 2 || third == 2)

count = count + 1;

end

end

prob = count/rolls

Output

>> statsWeek2

prob =

0.4212

Explanation

I first initialize two variables, the rolls variable determines how many iterations of the for loop below are performed, and the count variable acts as a running tally of how many rolls have at least one 2.

Within the for loop I initialise three variables first, second and third, each using the randi() function, by passing 6 to this function it returns a random integer between 1 and 6. I then check if any of the three variables are a 2 and if so I increment the count.

Once the loop is finished I divide the count by the total number of rolls to get the probability a roll had at least one 2. As you can see the output matches my answer to part (b).

1. {6, 6, 5} x 3 = 3 / 216 = 0.0138

There is only one sum of three dice that sums to 17: two 6s and a 5. As there are three positions each digit can be in, our one set of digits is multiplied by 3, giving us 3 elements in the sample space. We then divide this by the total number of elements in the sample space to get the probability.

1. 1 X X => 6 x 6 = 36 => {1, 5, 6} or {1, 6, 5} => 2 / 36 = 0.055

Given we know the outcome of the first dice roll, that leaves two dice uncertain, with 6 options each we get 6 \* 6 = 36 total permutations. In order for the total sum to be 12 the two dice with unknown outcomes must add to 11, there are only two ways this can occur, with a 5 and a 6 or vice versa. This gives us 2 permutations out of a possible 36, 2 / 36 = 0.055 .

Q2.

1. (5 / 6 x 1 / 20) + (1 / 6 x 1 / 6) = 0.0694

As the second dice outcome is 5 we cannot tell what the outcome of the first throw was so we must account for both eventualities.

The first case is when the first throw results in a 1, a 1 in 6 chance, in this case the second throw is with another six sided dice, so for it to result in a 5 has a 1 in 6 chance. In total for this case we have 1 / 6 \* 1 / 6.

The second case, where the first throw does not result in a 1, means we roll a 20 sided dice for the second throw. Therefore the probability of it resulting in a 5 is 5 / 6 \* 1 / 20.

To account for both eventualities we sum the probability of both outcomes.

1. (5 / 6) x (1 / 20) = 0.0416

As the outcome of the second throw is greater than 6 we can eliminate the scenario where the first throw is a 1. Meaning the probability of rolling a 15 is 5 / 6 \* 1 / 20.

Q3.

From Bayes Rule: P(E|F) = P(F|E)P(E) / P(F)

Where:

P(E) = the prior, the probability a person is guilty

P(F) = the probability a person has a characteristic

P(F|E) = the likelihood, the probability a person has a curtain characteristic given that they are guilty

P(E|F) = the posterior, the probability a person is guilty given that they have a certain characteristic - this is what we are looking for.

From the question we know:

P(E) = 60% = 0.6

The new piece of evidence tells us the guilty person has a certain characteristic and so

P(F|E) = 1

Since we know 20% of the population has a certain characteristic, we know the probability of a person having the characteristic, given they are not guilty is 20%

P(F|EC) = 0.2

We now have the three required variables to determine P(F) using the formula:

P(F) = P(F|E)P(E) + P(F|EC)(P(F|E)-P(E))

P(F) = 1(0.6) + 0.2(1 - 0.6) = 0.68

Then using Bayes Rule:

P(E|F) = P(F|E)P(E) / P(F)

P(E|F) = (1 \* 0.6) / 0.68 = 0.882

Q4. Code

PL = [0.05 0.1 0.05 0.05; 0.05 0.1 0.05 0.05; 0.05 0.05 0.1 0.05; 0.05 0.05 0.1 0.05];

PBL = [0.75 0.95 0.75 0.05; 0.05 0.75 0.95 0.75; 0.01 0.05 0.75 0.95; 0.01 0.01 0.05 0.75];

PB = [1 1 1 1; 1 1 1 1; 1 1 1 1; 1 1 1 1];

PLB = (PL.\*PBL)./PB

Output

>> statsWeek2

PLB =

0.0375 0.0950 0.0375 0.0025

0.0025 0.0750 0.0475 0.0375

0.0005 0.0025 0.0750 0.0475

0.0005 0.0005 0.0050 0.0375

Explanation

PL = matrix of probability of being at a certain location.

PBL = matrix of probability of having 2 bars of service given a certain location.

Since we are given the cell tower observation, we know the probability of having 2 bars at any location is 1, so

PB = matrix of probability of having 2 bars at a location.

Using Bayes Rule we can calculate PLB = (PL \* PBL) / PB.